Problem Solving for Introductory Physics

I. Make a clear diagram for the situation to be analyzed.
II. Form a mental image of the situation and imagine its time development if appropriate. Make predictions about the outcome of the problem. (Ex: The box will accelerate downward at (0 to <1) times g.)
III. Break complicated problems into a collection of simpler sub-problems. Solve the sub-problems and relate the results to solve the complete complex problem. Problems may be divided:
   a. To study each mass or particle individually. The several components of the motion of each particle and the relative motions on the several particles will be subject to constraints that must be expressed algebraically to reassemble the solution of the complex problem.
   b. Along the time line. The problem may be analyzed for the interval t_1 to t_2 and the end results used as initial conditions for the interval t_2 to t_3.
IV. Solve the resulting equations algebraically if possible using full vector notation where appropriate. Substitute the numerical data with associated units for each symbol in the result. In addition to the most precise statement that you can give for the result, give three significant figure answers with appropriate units and vector notation for problems with numerical data.
V. State your result in the language of the problem statement (without reference to coordinates that you have chosen, etc.).
VI. Compare the results with your expectations from II. What laws were used, and what did your learn about the content of those laws? Identify the mathematical techniques required for the solution. Master them.
VII. Check the units of your answer and any characteristic limits.

Step three of our general problem solving method.

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   b. Along the time line. The problem may be analyzed for the interval t_1 to t_2, and the end results used as initial conditions for the interval t_2 to t_3. It may be necessary to keep the results at the boundary times (like t_2) as algebraic expressions until the end of the calculation.

To be relevant, we will examine constant acceleration examples.

$$x(t) = x_o + v_o t + \tfrac{1}{2} a t^2$$
$$v(t) = v_o + a t$$
$$a(t) = a;\ \text{the constant acceleration}$$

The relations are valid only if the acceleration is constant over the entire time interval of interest. The values x_o and v_o are the position and velocity at the initial time, the beginning of the time interval, and t represents **the time that has elapsed since the particle was at the initial position traveling with the initial velocity**. Note that one need learn only the relation for x(t) which I refer to as the Master Equation for Constant Acceleration. The equations for v(t) and a(t) follow easily by taking time derivatives recalling that x_o, v_o and a are constant values.

Case I: Studying each particle or mass separately:
Treat as a 1-D problem. A passenger is behind a commuter train running at her maximum speed of 8 $^{m}/_{s}$ to catch a train. When she is a distance d from the rearmost entrance, the train begins to accelerate, starting from rest, at a constant 1.0 $^{m}/_{s^2}$ away from her. A.) Given d = 30 m, will she be able to catch up to be adjacent the entrance so she can jump on the train ?
B.) Make a graph showing position vs. time for the train and for the woman. Set x(0) = 0 for the train. C.) There is a critical value of d = d_c such that she can just barely get to the entrance; find it ! Look at your plot; what happens as d varies? You could have solved part A.) by using the quadratic formula. What happens in the quadratic formula approach when you have a critical situation? D.) For d_c, compare the woman's velocity with that of the train when she catches it. What was the train's average velocity between 0 and t_{catch} in this case?

Case II: Dividing a problem along the timeline.
A car initially at $x = 0$ accelerates from rest at a constant rate +A from t = 0 to t = t_I. At t_I, the driver begins braking at a uniform rate to give a constant acceleration of -2A. The car comes to a stop after traveling a total distance L in a total elapsed time of T. Find the distance that the car had traveled and the time when the brakes were applied.

Solve from t = 0 to t = t_I algebraically. $x(t_I) = {}^{1}/_{2}\, A\, t_I^{\,2}$ and $v(t_I) = A\, t_I$. Use the final values from the first time interval as the initial values for the next time interval: t_I. to t_F.. The expressions for velocity are computed as $^{dx}/_{dt}$. Use $v(t_F) = 0$.

Problem Solving for Compound Newton's Law Problems

I. Make a sketch and form a mental moving picture of the time development of the system.
II. Make a free body diagram for each mass **and** for any point or entity acted on by more than two forces. The free-body diagram for a body includes all the forces that act on that body and no others.
III. **For each** mass or entity chosen in II, choose coordinates (The axes must be perpendicular, and an axis should be chosen parallel to any component of the acceleration that you know. The acceleration perpendicular to an incline is usually zero - If true that would be a known.) and make a force table. Apply Newton's Second Law for each mass or entity chosen. Generate the Newton's Law equation for each component of the motion for each entity chosen.
IV. Add all equations of constraint applying to the motions of the individual masses such as $a_{1y} = 0$. Add the equations for the geometric constraints relating the motions of the several particles such as $a_{1x} = -\, {}^{1}/_{2}\, a_{2y}$. Express any other information available in the problem statement as an equation or initial (boundary) condition such as $v_{xo} = 3.0\ ^{m}/_{s}$. Also add supplemental relations based on Newton's third law or the equal magnitudes of the forces exerted at either end by ideal strings (tension) or struts (compression). In the cases that the strings or struts have mass, include free-body diagrams and Newton's Laws to analyze their motions.
V. Solve the equations using the Addition-Subtraction and Substitution Methods. When appropriate, I recommend substituting the constraint equation and then using addition-subtraction to eliminate the internal or common force. Additional algebraic and calculus techniques may be necessary. Complete the solution by substituting numeric data with units

for each known. Input information from friction models at this point after proposing that the contacting surfaces slip (kinetic) or do not slip (static) relative to one another.

VI. State the results in the language of the problem statement.

VII. Review the results and compare them with your expectations. What did you learn? What techniques were necessary to complete the problem solution? Pay special attention to the magnitudes of numerical results. Examine limits such as the largest and smallest meaningful angle. Develop an intuition for the sizes of forces, accelerations, etc. expected in various situations.

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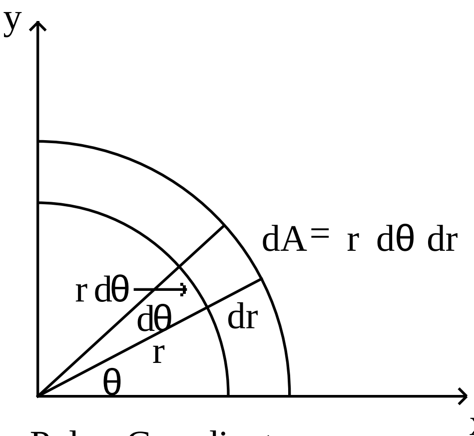
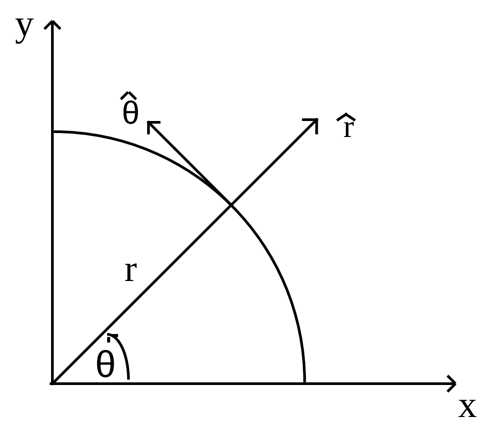
Circular Motion:



Polar Coordinates



The Coordinate Directions are the directions that the coordinate point moves when one coordinate is given a small positive increment while the other coordinates are held fixed. If only r is changed, the point moves radially away from the origin defining the radial direction $\hat{r}$. When q is changed by +dq, the point moves r dq [dq = 2p moves 2pr] counterclockwise around a circle of radius r. For small dq, the direction of the motion is along a tangent line to the coordinate circle at the point in the positive sense. This direction is the theta q or **tangential** direction at the point. Note: the radial and tangential directions change as the coordinate point changes. The Cartesian system is the only one for which the coordinate directions are the same for every point.

$$\hat{r} = \cos q \; i + \sin q \; \hat{j} \qquad \hat{q} = - \sin q \; i + \cos q \; \hat{j}$$

The line element in polar coordinates is the change in position when all of the coordinates are given small increments:

$$d\vec{\ell} = d\vec{r} = \vec{r}(r + dr,\ \theta + d\theta) - \vec{r}(r,\ \theta) = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

Dividing by dt, we see that the velocity is computed as $d\vec{r}/dt = dr/dt\,\hat{r} + r\,(d\theta/dt)\,\hat{\theta}$. To begin, we want to avoid integrating or differentiating directions like $\hat{r}$ and $\hat{\theta}$ which are not constants. Instead, we will use i, $\hat{j}$, and $\hat{k}$ which are constants.

$$\vec{r}(t) = R \cos(q)\; i + R \sin(q)\; \hat{j}$$

By computing time derivatives, we can find expressions for $\vec{v}(t)$ and $\vec{a}(t)$.

$$\vec{v}(t) = -\,{}^{d\theta}/_{dt}\, R \sin(q)\, i + {}^{d\theta}/_{dt}\, R \cos(q)\, \hat{j}$$
$$+ ({}^{dR}/_{dt})[\cos q\; i + \sin q\; \hat{j}\,]$$

For circular motion, ${}^{dR}/_{dt} = 0$.

$$v(t) = -\,{}^{d\theta}/_{dt}\, R \sin(q)\, i + {}^{d\theta}/_{dt}\, R \cos(q)\, \hat{j}$$
$$= {}^{d\theta}/_{dt}\, R\, [-\sin q\; i + \cos q\; \hat{j}]$$

$$\vec{a}(t) = -\,({}^{d\theta}/_{dt})^2\, R\, [\cos q\; i + \sin q\; \hat{j}\,] + (d^2\theta/dt^2)\, R\, [-\sin q\; i + \cos q\; \hat{j}]$$

Now that the derivatives are completed, we can express the results using $\hat{r}$ and θ .

$$\vec{v}(t) = {}^{d\theta}/_{dt}\, R\, \hat{\theta} \;=\; \omega R\, \hat{\theta} \quad \text{and} \quad \vec{a}(t) = \omega^2 R\, (-\,\hat{r}) + \alpha\, R\, \hat{\theta}$$

where we have defined $\omega = {}^{d\theta}/_{dt}$ and $\alpha = d^2\theta/dt^2$. It follows that $\omega = \pm\, {}^{v}/_{R}$ where the positive sign is for counter clockwise motion aroung the circular path.

For circular motion, the velocity must always be along a tangent to the circle as a radial component of the velocity would carry the particle off the circular path. The acceleration can have both radial and tangential components. The radial conponent is, in fact, negative or towards the center (centripetal) rather than away from the center (radial). The centripetal component of the acceleration has magnitude v^2/R where v is the speed of the particle and R is **the radius of the path**. The tangential component of the acceleration is just ${}^{dv}/_{dt}$, the time rate of change of the speed with counter clockwise motion chosen as positive by convention.

$$\vec{a} = {}^{v^2}/_{R}\, (-\,\hat{r}) + {}^{dv}/_{dt}\, \theta$$

How do you change a vector?

The magnitude of $\vec{v}$ is $v = \sqrt{\vec{v}\cdot\vec{v}}$, hence the rate of change of the magnitude:

$$\frac{dv}{dt} = \frac{\vec{v}\cdot\frac{d\vec{v}}{dt}}{\sqrt{\vec{v}\cdot\vec{v}}} = \left[\frac{d\vec{v}}{dt}\right]_{\parallel}$$, the component of $\frac{d\vec{v}}{dt}$ parallel to $\vec{v}$ *

*Dot product measures the degree to which two vectors are in the same direction.

The direction is $\hat{v} = {}^{v}/_{v}$ from which we compute

$$\frac{d\hat{v}}{dt} = \frac{\frac{d\vec{v}}{dt}}{\sqrt{\vec{v}\cdot\vec{v}}} - \frac{1}{2}\,\frac{2\,\vec{v}\cdot\frac{d\vec{v}}{dt}}{[\vec{v}\cdot\vec{v}]^{3/2}} = \frac{1}{v}\left\{\frac{d\vec{v}}{dt} - \left[\frac{d\vec{v}}{dt}\right]_{\parallel}\right\} = \frac{1}{v}\left[\frac{d\vec{v}}{dt}\right]$$

We conclude that a change parallel to the original direction of a vector changes its magnitude, and a change perpendicular to the original direction of a vector changes its direction.

Exercises: Let $\overline{A} = 100\, i$ and $\overline{B} = 200\, i$. Work only to first order in the Δ's.

1.) $\Delta\overline{A} = 0.5\,\hat{i}$, estimate ΔA and $\Delta\hat{A}$. Note $\Delta\hat{A}$ is not a unit vector!
2.) $\Delta\overline{A} = -0.5\,\hat{i}$, estimate ΔA and $\Delta\hat{A}$.
3.) $\Delta\overline{A} = 1.0\,\hat{j}$, estimate ΔA and $\Delta\hat{A}$.
4.) $\Delta\overline{B} = 1.0\,\hat{j}$, estimate ΔB and $\Delta\hat{B}$.

The change in a direction is always perpendicular to that direction. Why? What would a change parallel to a unit vector alter?

Inner Product of Vectors.

$\vec{A} \cdot \vec{B} = A\,B\cos\theta = A_x B_x + A_y B_y + A_z B_z$

This relation is particularly important as it is used to evaluate $v^2 = \vec{v} \cdot \vec{v} = v_x^2 + v_y^2 + v_z^2$.

The component definitions of the inner product can be used to evaluate the derivatives.

$$\frac{d\left(\vec{A} \cdot \vec{B}\right)}{dt} = \frac{d\left(\vec{A}\right)}{dt} \cdot \vec{B} + \vec{A} \cdot \frac{d\left(\vec{B}\right)}{dt} \text{ and } \frac{d\left(\vec{v} \cdot \vec{v}\right)}{dt} = 2\,\vec{v} \cdot \frac{d\left(\vec{v}\right)}{dt}$$

That is the product rule works for vector products. It works for the cross product as well as for the dot product.